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QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

NEW QUESTIONS.

The following interesting questions were formulated by Professor S. Kakeya at the request of the editors, and transmitted through Professor Birkhoff. Professor Kakeya has treated some problems of similar nature in various papers in the *Science Reports* of the Tôhoku Imperial University. They involve considerations of decided difficulty in the analytic formulation of geometric relations.

39. There are certain problems in geometry which are simple in statement but can be reduced only to very complicated problems in transcendental analysis. Following are several examples of the type of problem in question.

1. What is the smallest plane area within which a given figure can be turned through a complete revolution? It is not implied that the figure should revolve about a fixed point, but merely that in the course of its motion the figure should have every possible orientation in the plane. The problem may be modified by considering only convex areas.

An interesting special case is that in which the given figure is a segment of a straight line. In this case it has been conjectured by Professors Osgood and Kubota that the smallest area may be bounded by a three-cusped hypocycloid; if we consider only convex areas, perhaps the result will be an equilateral triangle. I have no indication of a proof.

2. For every closed convex curve of area P there is an n-sided circumscribed polygon of least area Q and an n-sided inscribed polygon of greatest area R. For a fixed value of the integer n and for all closed convex curves, what is the upper limit of Q/P and what is the lower limit of R/P? I have succeeded only in proving that for the case $n = 3$, the upper limit of Q/P is 2.

3. Let the area of a given simple closed curve A be a. Remove from A the greatest possible area a_1 similar to another given simple closed curve B. From the remaining figure remove the greatest possible area a_2 similar to B. Continue this process indefinitely. Is it or is it not true that

$$a_1 + a_2 + a_3 + \dots = a?$$

I have proved the statement to be true in the special case that A is convex and B is a circle.

4. Let a given closed convex curve K have the property that a given triangle whose angles are incommensurable with π can be revolved completely within K (see part 1 of this question), always remaining inscribed in K. What may the curve K be? Can any other curve except a circle satisfy the conditions?

DISCUSSIONS.

For a special type of quintic equation whose coefficients involve two parameters, Mr. C. B. Haldeman, in the first discussion following, obtains an algebraic solution by means of elementary operations and the extraction of roots; he also derives expressions for the roots in trigonometric form, and gives a graphical construction by the use of a certain cubic curve and a circle.

Professor Trevor, who has previously given instances of the use of mathematics in thermodynamics (*1919*, 444–447; *1920*, 55–57), presents an interesting set of results in connection with the study of a state of thermodynamic equilibrium in a homogeneous fluid. Several transformations of independent variables in a fundamental relation yield corresponding interpretations in the physical problem.

Professor Ransom gives a simple formula which may be used as a check in connection with the ambiguous case in the solution of plane triangles. Of course it applies only to problems which yield two solutions. Apparently there is a similar formula for the analogous case of spherical triangles:

$$\tan \tfrac{1}{2}(c' + c'') = \tan b \cos A,$$

and another for the dual case of ambiguity.

Professor Dick discusses a relationship between the dimensions of the "King's Chamber" in the Great Pyramid, the regular pentagon, and the regular icosahedron and dodecahedron. The implication of a connection between this relationship, the occurrence of the numbers of Fibonacci in plant life, and certain mystic statements of the ancient mathematicians will perhaps seem to most readers to be somewhat far-fetched. It should also be stated that the relation between a regular pentagon and the inscription of a square in a semicircle is clearly implied by the usual construction.

I. Resolution of a Certain Quintic Equation and a Geometrical Construction for its Roots.

By C. B. Haldeman, Ross, Butler County, Ohio.

1. The transformation

$$y = x - \frac{a}{x}$$

will reduce the equation $y^5 + 5ay^3 + 5a^2y + 2b = 0$ to $x^{10} + 2bx^5 - a^5 = 0$; from which we find x, and consequently

$$y = \sqrt[5]{- b + \sqrt{b^2 + a^5}} + \sqrt[5]{- b - \sqrt{b^2 + a^5}}.$$

If $b^2 + a^5$ be negative it appears the roots of the given equation are trigonometrical functions of the coefficients; for the transformation

$$y = - 2S\sqrt{- a}$$

will reduce the given equation to

$$16S^5 - 20S^3 + 5S = \frac{b}{a^2\sqrt{- a}};$$

and since

$$16 \sin^5 A - 20 \sin^3 A + 5 \sin A = \sin 5A,$$

we may take

$$S = \sin A, \qquad \frac{b}{a^2\sqrt{- a}} = \sin 5A$$

and get

$$y = - 2\sqrt{- a} \sin \tfrac{1}{5} \sin^{-1} \frac{b}{a^2\sqrt{- a}},$$